N71-28945

NASA TECHNICAL TRANSLATION · NASA TT F-13,614

SOLAR CORONAL PLASMA TURBULENCE SPECTRA AND THE STRUCTURE OF A RADAR SIGNAL. NO. 1.

I. M. Gordon, V. A. Liperovskiy, V. N. Tsytovich

Translation of "Spektry turbulentnosti solnechnoy koronal'noy plazmy i struktura radiolokatsionnogo signala.1," Report Pr-35, Institute for Space Research, Academy of Sciences, USSR, Moscow, 1970, 35 pages

CASE FILE COPY

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1971

SOLAR CORONAL PLASMA TURBULENCE SPECTRA AND THE STRUCTURE OF A RADAR SIGNAL. NO. 1.

I. M. Gordon, V. A. Liperovskiy, V. N. Tsytovich

ABSTRACT: Turbulence in the active regions of the solar corona is analyzed. A relatively limited problem, that of finding the distribution in the intensity of a reflected signal in terms of frequency shifts, and of comparing it with observations, is posed. The solution of this problem points to the possibility of posing a more general problem, that of determining the turbulence spectrum.

According to present day concepts, the solar corona is warmed by disturbances issuing from the convective zone. /1* These disturbances probably are magnetohydrodynamic shock waves with a solid front. Conditions for the occurrence of high-frequency turbulence, the dissipation of which is a direct source of heating of the corona, evidently are present at the fronts of waves such as these. Given this interpretation, the heating is turbulence heating, but the details of the mechanism involved in this heating are completely unknown. Indicative of corona turbulence are the numerous appearances of activity associated with sporadic radio emission, pointing to the turbulization of the plasma by fast beams. The possibility of detecting turbulence spectra is extremely important in attempting to explain the mechanisms of heating, and the processes taking place in coronal plasma.

Theoretical papers [1, 2], in which it is shown that the stationary spectra of Langmuir turbulence can be determined by the unique parameter Q, by a flow of turbulent energy along a spectrum that is equal, under quasi-stationary turbulence conditions, to the output power, have been published recently.

On the other hand, it was shown that the results of numerous radar investigations of the sun [3, 4] cannot be interpreted within the framework of the conventional theory of reflection from moving heterogeneities, but are the result of the interaction of the signal with the turbulence of the coronal plasma [5, 6, 7]. It is known that the scattering in turbulent plasma exceeds the scattering in undisturbed plasma by many orders of magnitude, as follows /2

---

* Numbers in the margin indicate pagination in the foreign text.

from nonlinear plasma theory [8, 9, 10], and as has been observed in many laboratory experiments [11, 12, 13].

It seems the natural thing to do to pose the problem of detecting the turbulence spectrum by using data on the frequency-time structure of a signal reflected by the sun. There is a basis for assuming that a properly made analysis of the frequency structure of a reflected signal will make it possible to restore the picture of plasma turbulence in the scattering layers of the solar corona.

There are a great many data from observations that definitely point to the fact that the characteristics of coronal turbulence are different in different zones of the corona. Sporadic radio emissions of various types, as well as powerful X-ray emissions, point to an elevated level of turbulence in the corona over the active regions. Analysis of the data from radar investigations of the sun have shown that the basic components of a signal reflected from the sun are formed in the pillar of the active corona resting on the active regions (floccular fields) [7]. Therefore, the analysis that follows will relate to the turbulence in the active regions of the solar corona.

At this point, it should be noted that not all types of turbulent pulsations can make any marked contribution to the formation of a reflected signal in the direction to the observer. As a matter of fact, change in the total pulse during scattering is $2\hbar\vec{k}$, if frequency shifts are ignored, so during scattering a like pulse should be carried away by turbulent pulsations. Since, of necessity, the transmitted frequency is $\omega \gg \omega_{pe}$, the possibility of scattering by turbulent pulsations with $k < \omega_{pe}/c$ is precluded immediately. It is known that when $k > \omega_{pe}/c$, only high-frequency electron pulsations are possible in plasma, specifically Langmuir pulsations, as well as ion-sound pulsations. These pulsations make three-plasma and four-plasma scattering possible. /3

Both the four-plasma, and the three-plasma processes give identical frequency shifts $\Delta\omega \sim \omega v_s/c$ ($v_s = \sqrt{T_e/m_i}$, $T_e \gg T_i$), for scattering in the opposite direction, so far as ion-sound pulsations are concerned. Four-plasma processes of this type therefore always are less significant than are the three-plasma ones. The frequency shift $\Delta\omega \sim \omega_{pe}$ is far in excess of that observed in the experiments [3, 4] in the case of Langmuir waves during three-plasma processes.

Therefore, four-plasma scattering by Langmuir waves must be taken into consideration along with scattering by ion-sound waves. Four-plasma scattering such as this can yield small shift components, $\Delta\omega_{min} = 0$, and does not require the condition $T_e \gg T_i$; that is, it is possible in isothermal plasma. Although the absence of isothermality is the more probable case, given conditions of development of turbulence because of turbulent heating of the electrons, it must be borne in mind that turbulent heating is slow, and can be negated by other processes. Therefore, it can be submitted that scattering in isothermal plasma attributable to four-plasma processes is very substantial. /4

This paper will deal with a relatively limited problem, that of finding the distribution in the intensity of a reflected signal in terms of the frequency shifts, and to compare this result with observations. The solution of this problem will point to the possibility of posing a more general problem, that of determining the turbulence spectrum, that is, the distribution of the intensity of the Langmuir pulsations in terms of the turbulence scales from the experimental data on the distribution of the intensity of a reflected signal in terms of frequency.

1. Statement of the Problem and Basic Equations

An external signal from the earth and incident on the sun, will be considered as flat and monochromatic, in accordance with the data obtained by radar experiments with the sun.

$$W^t = W^o(z)\ \delta(\bar{k} - \bar{k}_o). \qquad (1)$$

Here, W^t is the electromagnetic wave energy density in the interval $d\bar{k}$ of signal wave numbers (the corresponding signal frequency is $\omega_o = k_o c$), while

$$\int W^t \, d\bar{k} = W^o(z),$$

where

$W^o(z)$ is the total signal energy per unit plasma volume[1].

---

1. The magnitude $W_{\bar{k}}$ can be expressed quite readily in terms of the spectral density $I_{\omega_1 e}$ of the electromagnet radiation equated to $d\omega$ and the solid angle $d\Omega$, most often used in astrophysics

The z coordinate expresses the distance from the level to the sun where $\omega_{pe} = \omega_o$, and is read in the direction to the earth. Then we shall omit it from the series of formulas, understanding, however, that the parameters of plasma turbulence are dependent on this coordinate. The index "o" here, and in what follows, will designate the frequency and wave vector of incident waves. /5

Attenuation of the original beam of radio waves, that is, W^o in terms of z, can occur during scattering by particles of the interplanetary plasma, as well as during scattering by waves of plasma turbulence. Moreover, attenuation also is possible as a result of the amplification process in a turbulent plasma (the maser effect). This was reviewed in [10]. All of these effects are included in the $W^o(z)$ relationship as well. Since we are interested only in the "back" scattering, directly backward from the specified element of the volume of the solar plasma to the earth, the small solid angle /6

$$\Delta\Omega_1 = \frac{S_a}{R_0^2}, \qquad (2)$$

where

S_a is the area of the receiving antenna;

R_o is the distance from the sun to the earth,

corresponds to this scattering.

Let $Y\omega_1 e$ be the spectral power of the radiation from a unit of volume of the plasma for a backwardly scattered wave, equated to unit solid angle. Then the amplitude

$$I_{\omega,\Omega}\Delta\Omega\Delta z S_1 2\pi\Delta\nu_{box}$$

characterizes the experimentally obtained power of the reflected radiation equated to the "box" in the spectra in [5, 6].

Here $\Delta z = 7\cdot10^9$ cm^2, S_1 is the cross section of the plasma "column" in the active zone on the sun, $\Delta\nu_{box}$ is the resolution frequency interval ($\Delta\nu_{box} = 5$ kHz).

$$W_k dk = W_{\omega,\Omega} d\omega d\Omega,$$

$$dk = d\Omega d\omega \frac{\omega^2}{c^3}, \text{ that is,}$$

$$W_{\omega,\Omega} = W_k \frac{c^3}{\omega^2}.$$

The process $t + l \to t' + l'$ determines the magnitude of the power with which the "backwardly" scattered radiation is generated in the case of isothermal plasma ($T_e = T_i$). Taking it that the spectral density of the energy of the backwardly scattered waves is much less than that transmitted from the earth, it can be supposed (see [8, 5]) that the intensity of the scattered signal can be determined only on the basis of the incident energy density $W_{\vec{k}}^{t}$ and of the turbulence energy density $W_{\vec{k}}^{l}$

$$I_{\omega,\Omega} = \int d\vec{\kappa}_1 W_0(\kappa) \alpha(\omega, \Omega, \vec{\kappa}_1, \vec{\kappa}_2) W_{\vec{\kappa}_1}^{l} W_{\vec{\kappa}_2}^{l} d\vec{\kappa}_2 \tag{3}$$

$$W^{l} = \int W_{\vec{\kappa}}^{l} d\vec{\kappa} \tag{4}$$

energy density l for waves in 1 cm^3

$$\alpha = \frac{\omega^2}{(2\pi)^3 c^3 \omega_{pe}^2} w_{tl}^{t'l'} \tag{5}$$

$$\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e}, \quad \lambda_e = v_e / \omega_{pe}, \quad v_e = \sqrt{T_e / m_e}.$$

The $W_{tl}^{t'l'}$ probability can be found in [8, 15]

$$w_{tl}^{t'l'}(\vec{\kappa}^t, \vec{\kappa}_1^l | \vec{\kappa}_2^l, \vec{\kappa}_3^t) = \frac{(2\pi)^9 e^4}{4(4\pi)^3 m_e^4 \omega_{pe}^2 \omega^2} \frac{(\vec{\kappa}_1 \vec{\kappa}_2)^2}{\kappa_1^2 \kappa_2^2} \frac{l}{\lambda_e^3} \times \left(1 + \frac{(\vec{\kappa}\vec{\kappa}_3)^2}{\kappa^2 \kappa_3^2}\right) \left| \frac{\mathcal{E}_i^l(\kappa_1 - \kappa_2)}{\mathcal{E}^l(\kappa_1 - \kappa_2)} \right|^2 \delta(\vec{\kappa}^t + \vec{\kappa}_1^l - \vec{\kappa}_2^l - \vec{\kappa}_3^t) \times \delta(\omega^t + \omega_1^l - \omega_2^l - \omega_3^t) \tag{6}$$

$$\kappa_1 - \kappa_2 \equiv \{\vec{\kappa}_1 - \vec{\kappa}_2, \omega_1 - \omega_2\} \text{ etc.}$$

Further calculations of scattered power depend on a concrete assumption as to the nature of turbulent motions. As a first step, we will assume local isotropy of the turbulence because of the effective processes involved in the development of isotropy [8, 16]. Although one generally cannot preclude definite

anistropy in the distribution of pulsations when magnetic fields are present, we will begin with a consideration of the very simplest case of isotropic turbulence. This will make it possible to integrate Eq. (3) in terms of the directions in which the turbulent pulsations are distributed. However, we must first analyze qualitatively the laws of conservation in the scattering processes indicated, assuming that turbulent waves have arbitrary directions of distribution.

2. Analysis of the Laws of Conservation

Let us consider initially some of the conclusions following directly from the laws of conservation of energy and pulses for waves containing $t \to t' + s$ and $t + t \to t' + t'$ processes in the δ-functions of the corresponding probabilities, without making any assumptions as to isothermality, and in this way including scattering by ion sound.

We have, for the decay process $t \to t' + s$ (see Figure 1a)

$$\vec{K}_0 = \vec{K} + \vec{K}_s \qquad \text{and} \qquad \omega_0 = \omega + \omega_s \,. \tag{8}$$

Since $\vec{k} = -\vec{k}_s$, $2\vec{k}_0 = \vec{k}_s$ with sufficient accuracy, and consequently

$$\Delta\omega_{red} = \omega - \omega_0 = -\omega_s = -k_s v_s = \tag{9}$$

$$= -2k_0 v_e \sqrt{\frac{m_e}{m_i}} = -2\frac{\omega_{s0}}{c} v_e \sqrt{\frac{m_e}{m_i}} \equiv -\Delta\omega'$$

Thus, when the decay process involves ion-sound waves, we obtain a red shift by a completely determined magnitude. The width of this shifted line is determined by the width of the transmitted signal spectrum, and by the divergence in plasma temperatures in terms of time and space.

In a completely analogous way we obtain the following for the confluence process

$$\vec{K}_0 + \vec{K}_s = \vec{K} \qquad \omega_1 + \omega_s = \omega \tag{10}$$

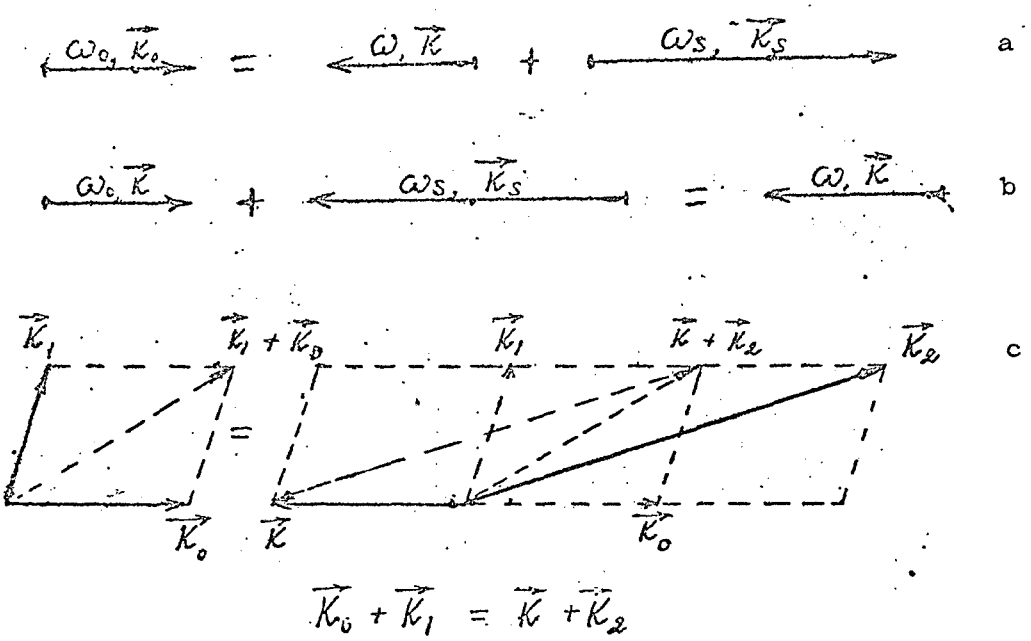

Figure 1.

Since $\vec{k} = -\vec{k}_o$, $2\vec{k}_o = \vec{k}_s$ with sufficient accuracy, and consequently,

$$\Delta\omega_{violet} = k_s v_s = 2k_o v_e\sqrt{\frac{m_e}{m_i}} = 2\frac{\omega_o}{c}v_e\sqrt{\frac{m_e}{m_i}} \equiv \Delta\omega' \qquad (11)$$

which corresponds to the Eq. (9) violet shift.

The laws of conservation for the four-plasma process of scattering by Langmuir pulsations (the subscripts 1 and 2 correspond to the Langmuir waves)

$$\omega_o + \omega_1 = \omega + \omega_2 \qquad \vec{k}_o + \vec{k}_1 = \vec{k} + \vec{k}_2 \qquad (12)$$

yield

$$2\vec{k}_o = \vec{k}_2 - \vec{k}_1 \text{ and}$$

$$\Delta\omega = \frac{6\omega_e^2}{\omega_{pe}}\left(k_1^2 - \vec{k}_1\vec{k}_2\right) \qquad (13)$$

when $\vec{k} = -\vec{k}_o$.

As will then be seen from Eq. (13), the reflected signal is a continuum of some width because of the four-plasma processes $t + t \to t' + t'$.

The maximum frequency shift during four-plasma scattering can have the value

$$\Delta\omega_{max} = 6\omega_{pe}\kappa_0^2\lambda_e^2 \equiv \Delta\omega'' \tag{14}$$

Let us note that the following inequalities occur in both cases (because the turbulent pulsations should track the region of transparency

$$\kappa_g = 2\kappa_0 = 2\frac{\omega_0}{c} < \frac{\omega_{pe}}{v_e} = \frac{1}{\lambda_e} \tag{15}$$

and

$$|\vec{\kappa}_1^l - \vec{\kappa}_2^l| = 2\kappa_0 < 1/\lambda_e \tag{16}$$

from whence $\omega_0/c < 1/2\lambda_e$

On the other hand, $\omega \gg \omega_{pe}$, so

$$\omega_{pe} \ll \omega < \omega_{pe}\frac{c}{2v_e} \tag{17}$$

This is a mandatory condition for a signal frequency at which "backward" scattering can take place during the $t \to t' + s$ and $t + t \to t' + t'$ processes. /11

Let us pause to consider yet another important inequality limiting the region of action of the processes studied.

Let us suppose that k_0 is such that

$$\kappa_0\lambda_e \leq \frac{1}{3}\frac{v_i}{v_e}$$

Then

$$\omega_0 \leq \frac{1}{3}\frac{v_i c}{v_e^2}\omega_{pe}$$

On the other hand, $\omega_{pe} \ll \omega$, and therefore

$$\omega_{pe} \ll \omega_0 \leq \frac{1}{3}\frac{v_i c}{v_e^2}\omega_{pe} \tag{18}$$

An interval such as that of Eq. (18) can exist if $v_i c \gg 3v_e^2$; $v_e \ll 2\cdot 10^8$ cm/s for isothermal plasma (or $T_e \ll 2.5\cdot 10^{50}$).

Since this temperature is much too low for the solar corona, in what follows we shall take it that

$$\frac{1}{3}\frac{v_i}{v_e} \ll \varkappa_0 \lambda_e \ll 1 \tag{19}$$

Given the Eq. (19) condition, it can be shown quite readily that the maximum frequency shift during the four-plasma processes (corresponding to an isolated satellite) always is greater than is the shift during scattering by ion-sound pulsations

$$\frac{\Delta\omega^{(3)}}{\Delta\omega''} = \frac{1}{3}\sqrt{\frac{m_e}{m_i}}\frac{1}{\varkappa_0\lambda_e} \ll 1 \tag{20}$$

At the same time, as distinguished from ion-sound waves, Langmuir waves provide /12
satellite, as well as only slightly shifted components.

3. Shape of the Central Part of the Spectrum of Scattering in an Isothermal Plasma

If Langmuir waves are excited directly, only Langmuir turbulence will occur in the initial stage, given plasma isothermality $T_e = T_i$, and this is true in all cases. There can be no ion-sound waves in this case. We will limit ourselves to this case in what follows in this article.

As has been shown [2, 17], when quasi-stationary sources of turbulence of power Q are present in the region of large wave numbers, the quasi-stationary spectrum of Langmuir waves has the form shown in Figure 2.

Here $W_{\varkappa} = W^l_{\varkappa} 4\pi\varkappa^2$ is the spectral density of the energy of the Langmuir waves in the isotropic case with normalization

$$\int W^l_{\varkappa} d\varkappa = W^l$$

(k is the modulus of $\vec{k}$), W^l is the total energy of the Langmuir turbulence per unit volume, $\varkappa_* = \frac{1}{3\lambda_e}\sqrt{m_e/m_i}$, $\varkappa_0^l = 1/L_0$ is the fundamental scale for turbulence, established by the power of generation, Q (or the total energy of the Langmuir waves, equal to $W^l = 2Q/\nu_e$, under quasi-stationary conditions)

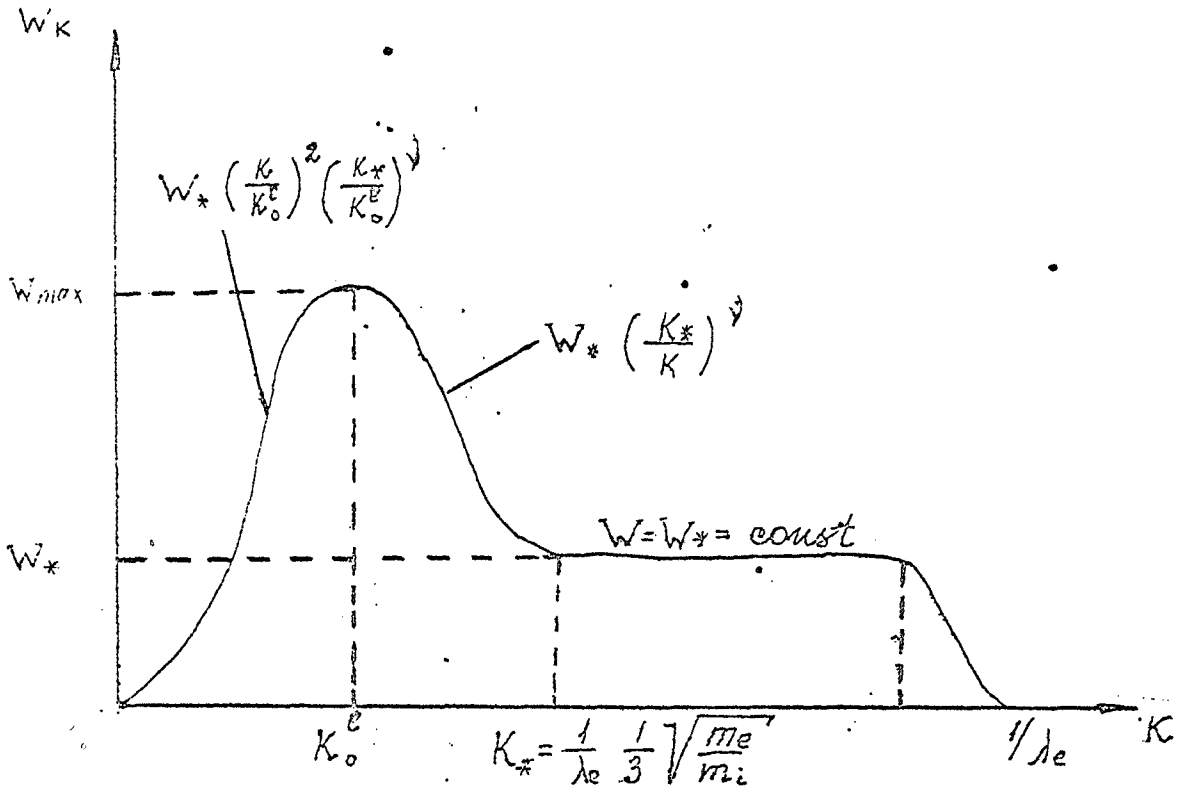

Figure 2.

$$K_0^l = K_* \left(0.1 \frac{\nu_e^2}{\omega_{pe}} \frac{nT_e}{2Q}\right)^{\frac{1}{2(\nu-1)}} = \tag{21}$$

$$= K_* \left(0.1 \frac{\nu_e}{\omega_{pe}} \frac{nT_e}{W^l}\right)^{\frac{1}{2(\nu-1)}}$$

$$(2.34 \leq \nu < 4)$$

/13

The magnitude $W_* = \sqrt{2Q/\alpha}$, where $\alpha = \frac{\pi \omega_{pe}^3}{4.27 m_i n_0 \nu_e^4}$

from whence

$$\frac{W_*}{nT_e \lambda_e} = \sqrt{\frac{W^l}{nT_e} \frac{\nu_e}{\omega_{pe}} \frac{10\Sigma}{\pi} \frac{m_i}{m_e}} \tag{22}$$

The function W_k can be approximated by the expressions

$$W_\kappa = \begin{cases} \left(\frac{\kappa}{\kappa_0^l}\right)^2 \left(\frac{\kappa_*}{\kappa_0^l}\right)^\nu W_* & \text{when} \quad 0 < \kappa < \kappa_0^l \\ W_* \left(\frac{\kappa_*}{\kappa}\right)^\nu & \text{when} \quad \kappa_0^l < \kappa < \kappa_* \\ W_* = \text{const} & \text{when} \quad \kappa_* < \kappa \ll 1/\lambda_e \end{cases} \tag{23}$$

This approximation is adequate for our purposes. A precise W_k, obtained as a result of a numerical computation made in [18] is extremely close to the values provided by Eq. (23). Accordingly, in the simplest of cases, that when $T_e = T_i$, ∠14 if we substitute the Eq. (6) probability in Eq. (3) for $I_{\omega_1\Omega}$, we obtain the following after integration with respect to the angles of distribution of the intensity of the scattered back radiation in terms of the frequency shifts

$$I_{\omega,\Omega} = \frac{W_0(\Omega) c \omega_{pe}}{\omega_0^4} \frac{\pi^3 v_e^2}{3 \cdot 2^2} \gamma(\Delta\omega) \tag{24}$$

Here $\gamma(\Delta\omega)$, determining the spectrum of scattered back radiation, can be expressed by the formula ($\Delta\omega = \omega - \omega_0$, ω_0 is the frequency of the incident signal)

$$\gamma = \left|\frac{\mathcal{E}_i^l(2\kappa_0, \Delta\omega)}{\mathcal{E}^l(2\kappa_0, \Delta\omega)}\right|^2 \int \frac{W^l(\kappa\lambda_e)}{nT_e\lambda_e} \cdot \frac{W^l\left(\lambda_e\sqrt{\kappa^2 + \frac{4\Delta\omega\omega_{pe}}{6v_e^2}}\right)}{nT_e\lambda_e} \times \tag{25}$$

$$\times \frac{d(\kappa\lambda_e)}{(\kappa\lambda_e)^3} \frac{\left(\kappa^2 - 2\kappa_0^2 + 2\frac{\Delta\omega\omega_{pe}}{6v_e^2}\right)^2}{\left(\kappa^2 + 4\frac{\Delta\omega\omega_{pe}}{6v_e^2}\right)^2}$$

$$\kappa > \kappa_0 - \frac{\Delta\omega\omega_{pe}}{6v_e^2\kappa_0} \tag{26}$$

∠15

$$\kappa > -\kappa_0 + \frac{\Delta\omega\omega_{pe}}{6v_e^2\kappa_0}$$

The conditions of Eq. (26) determine the limits of integration in Eq. (25).

Let us pass on to the dimensionless variable

$$x = \frac{\Delta\omega}{\omega_{pe}}\sqrt{\frac{m_e}{m_i}} \qquad y = 2k_0\lambda_e$$

for purposes of convenience. At the same time, in accordance with Eq. (19),

1/3 $\sqrt{\frac{m_e}{m_i}} \ll y \ll 1$

Let us consider the function

$$f(x,y) = \left|\varepsilon^{\ell}_i(2k_0, \Delta\omega)\Big/\varepsilon^{\ell}(2k_0, \Delta\omega)\right|^2 = \left\{\frac{1 + \frac{T_e}{T_i}\frac{1}{y^2}\left[1 - Y_+\left(\frac{x}{y}\sqrt{\frac{T_e}{T_i}}\right)\right]}{1 + \frac{T_e}{T_i}\frac{1}{y^2}\left[1 - Y_+\left(\frac{x}{y}\sqrt{\frac{T_e}{T_i}}\right)\right] + \frac{1}{y^2}}\right\} \qquad (27)$$

(See [19] for the expression for the Y_+ functions, for example). When $T_e = T_i$

$$f(x,y) = \begin{cases} 0.25 & \text{when } x \leq y \\ (x/y)^4 & \text{when } x \gg y \end{cases} \qquad (28)$$ /16

Since this investigation is being made for purposes of interpreting concrete experiments, let us note that $x \ll 1$ is always the case for these concrete experiments ($x \leq 1$ corresponds to a frequency shift in the scattered backward signal of $\Delta\nu = 0.45 \cdot 10^6$ Hz when the carrier frequency is $\nu = 38 \cdot 10^6$ Hz).

Now, for purposes of determinancy, let us consider the case when $\Delta\omega > 0$ (the violet part of the spectrum). In the case of $\Delta\omega < 0$ a similar consideration will give the red part of the spectrum symmetrical to the violet part.

If $x \leq y$, then $f(x, y) = 0.25$. Then the relationship is

$$\frac{x}{y^2\frac{3}{2}\sqrt{\frac{m_i}{m_e}}} \leq \frac{y}{y^2\frac{3}{2}\sqrt{\frac{m_i}{m_e}}} = \frac{2}{3}\frac{\sqrt{m_e/m_i}}{y} \ll 1 \qquad (29)$$

from whence, substituting $x = \Delta\omega\sqrt{m_i/m_e}/\omega_{pe}; y = 2k_0\lambda_e$ we obtain the inequality

$$\frac{\Delta\omega}{6\omega_{pe}} \ll (k_0 \lambda_e)^2 \tag{30}$$

Therefore, in the Eq. (25) integral the first approximation can ignore $\Delta\omega/\sigma\omega_{pe}$ as compared with $(k_o \lambda_e)^2$ and we have, changing for purposes of convenience to the dimensionless argument $\xi = k \lambda_e$

$$\mathcal{J}_{\dot{W}}(\Delta\omega) = \frac{1}{4} \int_{y/2}^{1} \left(\frac{\bar{W}^1(\xi)}{nT_e\lambda_e}\right)^2 \frac{d\xi}{\xi^3} \left(\frac{\xi^2 - \frac{1}{2}y^2}{\xi^2}\right)^2 = \tag{31}$$

$$= \overline{2\pi\omega_{pe} n_0 T_e m_e y^2}$$

that is, the intensity of scattering is totally determined by the parameter Q.

Here $W^1(\xi) = W_* = \text{constant}$, because the integration is made in terms of $\xi = k\lambda_e > k_* \lambda_e$ (see Figure 2 and Eq. (23)). The upper limit in the integral is replaced by 1, and this will introduce what obviously is an insignificant error. We see from Eq. (31) that the spectrum of the backwardly scattered radiation is almost constant up to the value

$$\Delta\omega \leq \Delta\omega' \simeq 2k_0 v_e \sqrt{m_e/m_i} \tag{31a}$$

corresponding to $x_{max} = y = 2k_0\lambda_e$

When $\Delta\omega > \Delta\omega'$ there is a sharp drop in intensity, determined by the decrease in the function $f(x, y)$ when $x/y > 1$.

The results when $\Delta\omega < 0$ are symmetrical to the results when $\Delta\omega > 0$, as similar computations have shown.

The almost constant section of the backwardly scattered spectrum (the "wing"), the intensity of which is established through Eq. (31), is shown in Figure 3.

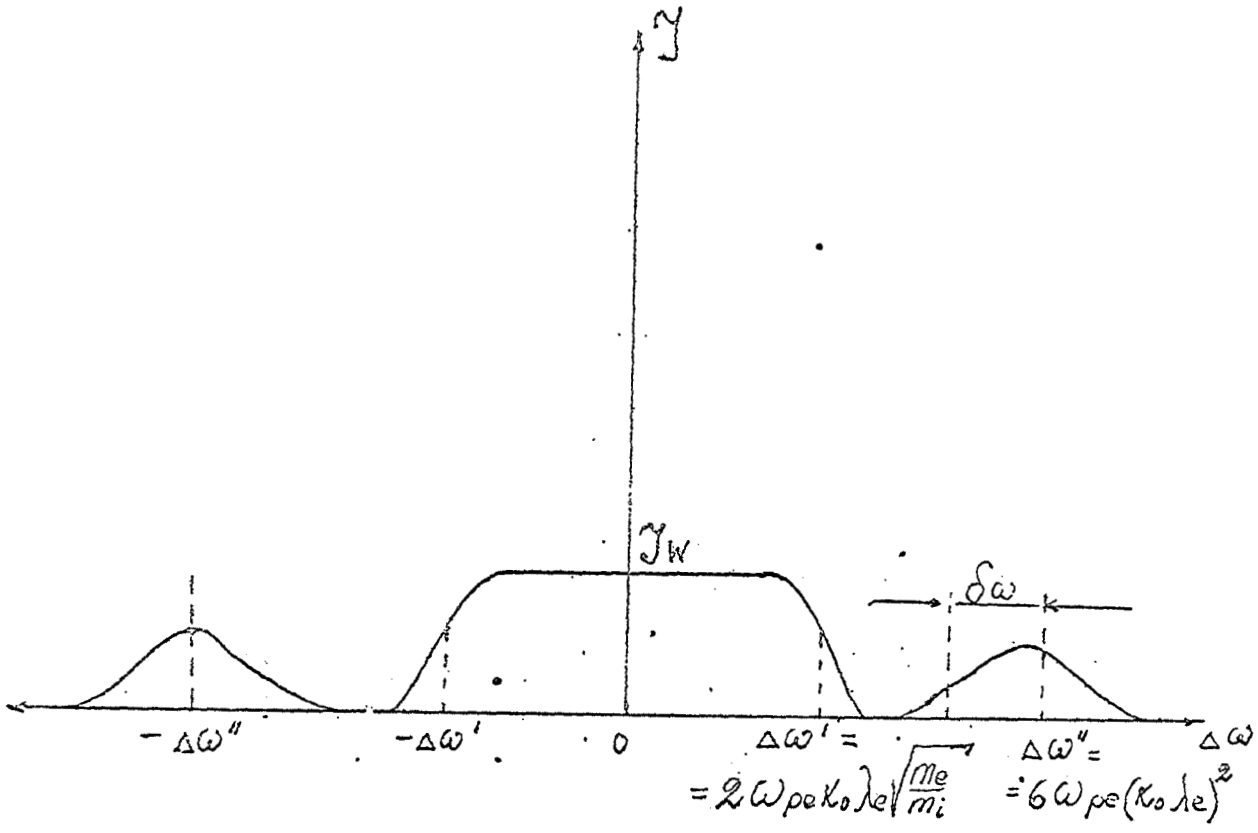

Figure 3.

We note that over the relative width of the "wings" of the spectrum

$$\frac{\Delta\omega'}{\omega} = \frac{2v_e}{c}\sqrt{\frac{m_e}{m_i}} = \frac{2}{c}\sqrt{\frac{T_e}{m_i}} \qquad (31b)$$

$$T_e = 1.8 \cdot 10^{-3}(\Delta V)^2$$

and can be used to judge plasma temperature.

4. Satellites Occurring When $t + t \to t' + t'$ in the Scattering Process

The case when $x \leq y$ we considered in the preceding section of this article permitted us to find only that part of the spectrum of backwardly scattered waves corresponding to comparatively small $\Delta\omega \leq \Delta\omega' \doteq 2k_0 v_e \sqrt{m_e/m_i}$. At the same time, the spectrum cut off by the function $f(x, y)$ is sharply incident (as (y/x^4) when $\Delta\omega > \Delta\omega'$. However, in this case the Eq. (25) integral characterizing the spectrum was established by the spectral function of the Langmuir turbulence W_k^t when $k > k_*$, where the function $W_k^t = W_* \ll W_{max}$; that is, at a distance from the maximum corresponding to the fundamental turbulence scale, k_o^t.

There is, therefore, another case that can be considered, that of $x/y \gg 1$, in which $Y(\Delta\omega)$ is not small, despite the fact that $f(x, y) = (y/x)^4$, and one will yield a small factor to the Eq. (25) integral. This situation can occur if the integral itself is great because the spectral function of the Langmuir turbulence near its maximum contributes to it. /19

As a result of this consideration, there will be two symmetrical (red and violet) satellites in addition to the wings mentioned, and the centers of these satellites will be distant from 0 by $\Delta\omega' = \pm\sigma\omega_{pe}(k_o)$.

Let us investigate in detail the structure of the satellite line within the limits of the formula for the left half of the violet satellite (see Figure 3).

Let $\Delta\omega = \Delta\omega'' - \delta\omega$, where $\Delta\omega''$ corresponds to the center of the satellite. Then the lower limit, ξ_{min}, in the Eq. (25) integral is determined by the first condition of Eq. (26) (the second is satisfied automatically)

$$\xi_{min} = k_0 \lambda_e - \frac{\Delta\omega}{6\omega_{pe}} \frac{1}{k_0 \lambda_e} = \frac{2}{y} \frac{\delta\omega}{6\omega_{pe}} \tag{32}$$

and we obtain

$$\gamma_{sat} = \gamma_{sat}(\delta\omega) = \frac{1}{x^4} \frac{W_*}{(nT_e\lambda_e)^2} \int_{\frac{2\delta\omega}{6\omega_{pe}y}}^{\xi_*} W^l(\xi)\xi^3 d\xi \tag{33}$$

The region from $\xi = \xi_{min}$ to $\xi = \xi_* = \frac{1}{3}\sqrt{m_e/m_i}$, where $W_k \gg W_*$, makes the basic contribution to the Eq. (33) integral, as the computations show. We /20 therefore take the value $\xi = \frac{1}{3}\sqrt{m_e/m_i}$ in place of 1 for the upper limit.

As will be seen from Eq. (33), the shape of the satellite line is uniquely linked with the spectral function of the Langmuir turbulence.

Differentiating Eq. (33) with respect to $\delta\omega$, we obtain

$$\frac{W^2\left(\frac{2\delta\omega}{\gamma 6\omega_{pe}}\right)}{nT_e\lambda_e} = -\frac{3\omega_{pe}\gamma}{\delta\omega}\,\frac{x^4 nT_e\lambda_e}{W_*}\,\frac{\partial\gamma_{sat}(\delta\omega)}{\partial(\delta\omega)} \qquad (34)$$

It is possible therefore, in principle, to find the spectrum of the Langmuir turbulence, by plotting the experimental curve of $\gamma_{sat}(\delta\omega)$.

It would be extraordinarily interesting to make an experimental check of these subtle effects associated with the delicate structure of the satellite line. To do so would nessitate increasing radio telescope resolution. Let us assess $\Delta\nu'' = \Delta\omega''/2\pi$ for the satellite. Let us say, by way of an example, that

$$\Delta\omega'' = 6\omega_{pe}(k_0\lambda_e)^2;\ k_0\lambda_e = 1/30;$$

$$\omega_{pe}/\omega_0 = 1/3;\ \omega_0 = 2\pi\nu_0;\quad \nu_0 = 38\ \text{MHz}$$

Then $\Delta\nu'' = \Delta\omega''/2\pi = 90$ kHz. Correspondingly, when $k_0\lambda_e = 1/60$, $\omega_{pe}/\omega_0 = 1/3$ $\Delta\nu = 22$ kHz. The width of the satellite line $\delta\omega'$ can be assessed quite readily /21 if the sharp reduction in the value of the Eq. (33) integral when $\xi_{min} \geq k_0^{-1}\lambda_e$ is taken into consideration

$$\frac{\delta\omega'}{6\omega_{pe}}\,\frac{1}{k_0\lambda_e} \simeq k'\lambda_e \qquad (35)$$

Naturally, the relationship

$$\frac{\delta\omega'}{\Delta\omega_e} = \frac{k'_e}{k_0} \ll 1$$

always is less than unity because $\lambda_e k_0' \ll \frac{1}{3}\sqrt{m_e/m_i}$, $\lambda_e k_0 \gg \frac{1}{3}\sqrt{m_e/m_i}$.

Since $k_0' \sim Q^{-1/2(\nu-1)}$ where Q is the power of the generation of turbulent energy of the Langmuir waves, k_0' decreases with increase in Q, and the satellite line becomes more and more acute as compared with its distance $\Delta\omega = 0$.

Let us rewrite Eq. (35) in more detail, gathering together the formulas already written, and assuming that $1/2\ (\nu - 1) = 0.27$ when $\nu = 2.84$

$$\frac{\delta\omega'}{\Delta\omega_c} = \frac{k_0'}{k_0} = \frac{1}{3}\sqrt{\frac{m_e}{m_i}}\frac{c}{\omega_0}\left(0.1\,\frac{\gamma_e}{\omega_{pe}}\,\frac{nT_e}{W'}\right)^{0.29} \qquad (36)$$

Readily seen from Eq. (36) is that, in principle, the width of the satellite can be used to determine the fundamental scale for the turbulence k_0' (based on the assumption of the quasi-stationary nature of the turbulence), uniquely associated with W'/nT_e, or with Q. /22

Now, let us compare satellite intensity, γ_{sat}^{max}, and the intensity of γ_w. Eq. (33) yields

$$\gamma_{sat}^{max} = \gamma_{sat}\Big|_{\delta\omega=0} = \frac{1}{\chi_{max}^4}\frac{W_*}{(nT_e\lambda_e)^2}\int_0^{\xi_*} d\xi\,\xi^3 W'(\xi)$$

for the satellite maximum. Substituting the concrete shape of the spectrum of Eq. (23) in this equation, we obtain

$$\gamma_{sat}^{max} = \frac{1}{\chi_{sat}^4}\left(\frac{W_*}{nT_e\lambda_e}\right)^2 \xi_*^2\left(\frac{\xi_*}{\xi_0}\right)^{\nu-2}\frac{\nu+2}{4(\nu-2)} \qquad (37)$$

where $2.84 \le \nu < 4$, $\xi_* = \frac{1}{3}\sqrt{m_e/m_i}\,W''/nT_e\lambda_e$ as determined through Eq. (22), $\xi_0 = k_0\lambda_e$ as determined through Eq. (21), and $\chi_{sat} = \frac{3}{2}\gamma^2\sqrt{m_i/m_e}$.

For the ratio $\gamma_{sat}^{max}/\gamma_w$, we obtain

$$\frac{y_{sat}^{max}}{y_W} = \frac{y^2}{x_{sat}^4}\,\frac{1}{6}\,\frac{m_e}{m_i}\,\frac{\nu+2}{\nu-2}\left(\frac{3\pi}{4}\left(\frac{\nu-1}{\nu+2}\right)^2\frac{\omega_{pe}}{\nu_e}\,\frac{W^l}{nT_e}\right)^{\frac{\nu-2}{\nu(\nu-1)}} = \qquad (38)$$

$$= \left(\frac{2}{3}\right)^4\left(\frac{m_e}{m_i}\right)^3\frac{1}{y^6}\left(\frac{1}{3}\,\frac{n\lambda_e^3}{\ln\Lambda}\,\frac{W^l}{nT}\right)^{0.23}$$

It was assumed that $\nu = 2.84$ in the latter equality. We can find W^l/nT (or $Q = \frac{\nu_e}{2}W^l$) from the experimentally obtained ratio Y_{sat}^{max}/Y_w, at least in principle, or, putting it another way, we can estimate Q regardless of the preceding method used to find Q. Let us, by way of an example, make several estimates.

If $y = 0.05 = 2k_o\lambda_e$, $n_o = 0.5\cdot10^7$ cm^{-3}, $\omega_o = 38.10^6\ 2\pi$ sec^{-1}, $\nu = 2.84$, $\omega_o = 2\omega_{pe}$, $T_e = 10^6$°K, $v_e = 4\cdot10^8$ cm/sec, $W^l/nT = 0.1$, we obtain $y_{sat}^{max}/Y_w = 0.04$.

In another case, if $y = 1/30$ when $T_e = 0.45\cdot10^5$, and the other parameters remain as above, then $y_{sat}^{max}/Y_w = 0.35$. /24

In concluding this section of the paper, we should point out that the computations we made show the symmetrical nature of the right and left halves of the satellite, as well as symmetry of the red and violet satellites (we have here considered in detail only the left half of the red satellite).

Thus the complete picture of the spectrum, as shown in Figure 3, consists of broad, symmetrical, wings and satellites associated with the four-plasma scattering $t + l \to t' + l'$. At the same time, there always is a gap between the ends of the wings and the satellites.

5. Discussion of the Results of the Radar Experiments

We shall, in this section, attempt to make a comparison between the experimental spectra of a 38 MHz radar signal reflected from the sun [3, 4] and the results of scattering theory based on the four-plasma processes of the $t + l \to t' + l'$ type discussed in the preceding sections. At the same time, it must be remembered that the published spectra were obtained as a result of the accumulation

of the reflected signal for a period of ~1,000 seconds, an interval that corresponds to double the time required for a signal to cover the distance from the sun to the earth. This is significant, because there are definite indications that the spectrum of the turbulent pulsations of coronal plasma above the flocculi is not stationary for the duration of the observations. This is confirmed by the correlation between the elevated level and the fluctuations of the sporadic radio emission from the sun and the large effective diameters invariably observed. /25

Since sporadic radio emission is associated with the conversion of plasma waves into electromagnetic waves, this indicates an intrinsic transformation of the spectrum of plasma pulsations in a period of time of the order of seconds. Scattering of turbulized plasma in an unbalanced spectral distribution can make a significant contribution to the formation of the reflected signal. An increase in resolution to 1 second during the period of time of the radar experiments opens up the most interesting possibilities for making a direct investigation of the evolution of plasma spectra in terms of the change in the nature of the reflected signal spectrum.

A second comment concerns the problem of reflected signal intensity. The high value of the effective diameter, as high as $100R_{\odot}^2$, [21], imposes great demands on the effectiveness of the scattering in the backward direction. These demands are intensified even further by the fact that the reflection does not involve the entire corona, but only those parts of it that are located above the flocculi in the central part of the disk [7].

There are, in principle, two possibilities involved in explaining the results of the observations. The first is associated with the anisotropic nature of the scattering by the primary scattering in the direction of signal incidence, something that can occur when there are definite spectral distributions of the turbulent pulsations [6]. The other possibility is associated with signal amplification in the corona as a result of the processes of induced conversion of the Langmuir waves into electromagnetic waves. /26

Analysis of the observed data indicate that in fact both of these factors are involved.

It is possible to separate these two effects once radar experiments are set up to tract the sun continuously for a long period of time, something that requires equipping the antennas with electrical control of the antenna patterns.

Since this article will not concern itself with questions dealing with signal amplification, the discussion that follows will deal soley with spectrum shapes.

1. Reference [7] notes that all published spectra of the 38 MHz signal reflected from the sun can be broken down into two types. The first of these, Type A, has most of its components at a distance of $1.4R_\odot$ in the direction to the observer. Spectra of this type can be observed when there are calcium flocculi in the central part of the disk of the sun, and the higher values of effective diameter correspond to them. The Type B spectra have diffused structure. The individual components occur at different levels, and it is difficult to localize them because of the inadequate resolution realized in the experiments on 38 MHz [3, 4].

It is possible to estimate the mean temperature, T_e, proceeding from the halfwidth of the Type A spectra and utilizing Eq. (31b). These estimates are in good concordance with the available data on the corona, $T_e \sim 2\cdot10^6$°K. It is interesting to note that the higher effective diameters correspond to the higher temperatures and velocities (the spectra are compared in [3, 4]). An interesting feature of spectra structure, whether averaged for length of time intervals, or individually (the latter correspond to the higher effective diameter values), is the absence of a signal from the satellites among the basic components. Assuming that instead of signal formation there are standard conditions $T_e = 2\cdot10^6$k, $n_e \sim 10^7$ cm^{-3}, we have $\Delta\nu \leq 100$ kHz. Frequency shifts such as these were detected in experiments in recent years [3, 4]. It is possible that the absence of these satellites is associated with their low relative intensity. /27

With a slight conversion Eq. (38) can be presented in the form

$$\gamma = \frac{\gamma_{sat}^{max}}{\gamma_w} = 9.5\cdot10^{-5} n_e^{2.885} T_e^{-2.665} \beta^{0.23} \qquad (39)$$

where $\beta_0 = W^l/nT_e$. What follows is that when $T_e \sim 2 \cdot 10^6$°K, $n = 10^7$, $\beta = 10^{-4}$, we obtain $\gamma = 0.03$.

The satellites will be even weaker at higher temperatures. It is possible that the explanation for this is the absence of satellites in the spectra of the Type A reflected signal. Rapid growth in the velocity of the coronal plasma, /28 accelerating over the active regions, sometimes is observed [7]. This process is accompanied by a rapid decay in the concentration of the plasma, and by a rise in its temperature [22].

It is of interest to follow the transformation of the character of the spectrum attributable to scattering as a result of the four-plasma processes with increase in the distance of the scattering layer from the center of the sun. The width of the central part of the spectrum increases $\sim \sqrt{T_e}$, and when there is an increase in the temperature by a factor of 5, the latter increases by a factor of ~2.2. At the same time, the "center" of the satellites $\Delta\nu$ (sat) $\sim T_e/\sqrt{n_e}$ increases by a factor of ~12. Thus, spectra, the widths of which increase rapidly with height, and which are characterized by even more rapid increase in satellite shift, should correspond to cases of rapid acceleration in the solar wind over the active regions. Satellite intensity decreases rapidly with drop in concentration and rise in temperature. Thus, Type A spectra are those most probably detectable in the lowest layers of the corona.

This analysis has limited value because all that was considered was scattering in isotropic, isothermal plasma. It is proposed that consideration of the case of non-isothermal plasma, when scattering by ion-sound waves can play a significant part, and when the possible effect of the magnetic field is taken into consideration, be the subject of subsequent papers.

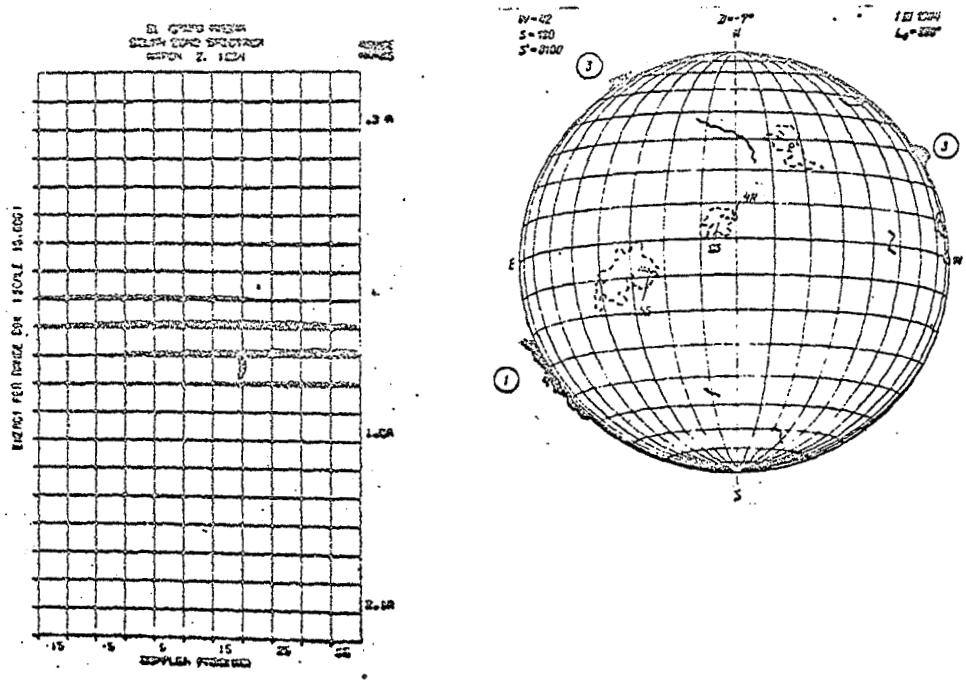

Figure 4. Type A spectra.

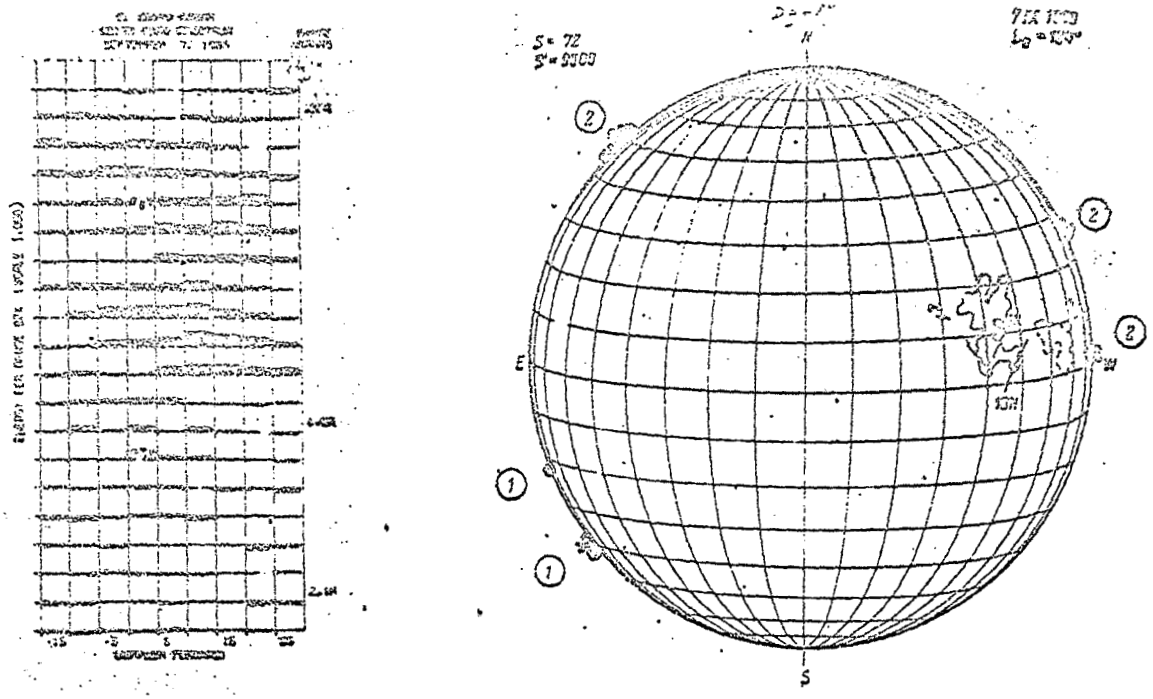

Figure 5. Type B spectra.

REFERENCES

1. Pikel'ner, S.B., Tsytovich, V.N., ZhETF, 55, 1968, p. 977. /29
2. Liperovskiy, V.A., Tsytovich, V.N., ZhETF, 57, 1969, p. 1952.
3. James, F.C., IEEE Transactions AP-12, 1964, p. 876.
4. James, F.C., Astrophys. J., 146, 1966, p. 356.
5. Gordon, I.M., Astron. Zh., 45, 1968, p. 1002.
6. Gordon, I.M., Astrophys. Lett., 2, 1968, p. 49.
7. Gordon, I.M., Astrophys. Lett., 3, 1969, p. 181.
8. Tsytovich, V.N., Nelineynyye protsessy v plazme [Nonlinear Processes in Plasma], Moscow, 1962.
9. Sakhokija, D.M., Tsytovich, V.N., Nuclear Fusion, 8, 1968, p. 241.
10. Kaplan, S.A., Tsytovich, V.N., UFN, 97, 1969, p. 77.
11. Demidov, B.A., Yelagin, N.I., Fanchenko, S.D., Pis'ma v ZhEFT [Letters to ZhEFT], 2, 1965, p. 12, 533.
12. Ascoli-Bartoli, V., Katzenstein, I., Lovisette, L., Nature, 204, N 4959, 1964, p. 672.
13. Hamberger, S., Preprint, Callam., 1968.
14. Kovrizhnykh, V.A., ZhEFT, 49, 1965, p. 1332.
15. Liperovskiy, V.A., Tsytovich, V.N., Izv. VUZov "Radiofizika," 9, 1966, p. 513.
16. Gudkova, V.A., Liperovskiy, V.A., PMTF, No. 3, 1970. /30
17. Pikel'ner, S.B., Tsytovich, V.N., Astron. Zh., 46, 1969, p. 8.
18. Tsytovich, V.N., Obzornyy doklad na 9-oy Mezhdunarodnoykonferentsii po yavleniyam v ionizirovannykh gazakh [Report to the 9th International Conference on Phenomena in Ionized Gases], Budapest, 1969.
19. Silin, V.P., Rukhadze, A.A., Elektromagnitnyye svoystva plazmy i plazmopodobnykh tel [Electromagnetic Properties of Plasma, and Plasma-Like Bodies], Moscow, 1961.
20. Liperovskiy, V.A., Tsytovich, V. N., PMTF, No. 5, 1965, p. 15.

21. Eshlaman, R.V., Science, 158, 1967, p. 11.

22. Bisnovaty-Kogan, G.S., Gordon, I.M., Astrophys. Lett., 4, 1969, p. 149.

Translated for the National Aeronautics and Space Administration
under contract No. NASw-2038 by Translation Consultants, Ltd.,
944 South Wakefield Street, Arlington, Virginia 22204.